Oblique electrostatic modes in self-gravitating dusty plasmas

Frank Verheest
Sterrenkundig Observatorium, Universiteit Gent, Krijgslaan 281, B-9000 Gent, Belgium

Victoria V. Yaroshenko
Centrum voor Plasma Astrofysica, Katholieke Universiteit Leuven, Celestijnenlaan 200^B, B-3001 Leuven, Belgium

(Received 4 October 2001; published 6 March 2002)

The propagation of oblique and perpendicular electrostatic modes in dusty self-gravitational magnetized plasmas is treated with due care for the small gravitational effects between charged dust particles and for the correct balance between electrostatic and gravitational forces. At purely oblique propagation, generalizations of the dust-cyclotron and dust-acoustic modes are found, where the latter can be subject to a Jeans instability. For strictly perpendicular propagation, only a mixed dust-acoustic and dust lower-hybrid mode occur at low frequencies. A Jeans collapse can be avoided by strong enough magnetic fields, the criterion for which is given.

DOI: 10.1103/PhysRevE.65.036415 PACS number(s): 52.27.Lw, 52.35.Fp, 95.30.Qd

I. INTRODUCTION

Dusty plasmas are characterized by the presence of charged dust particles, besides the usual electron and ion species. Charged dust can accumulate charges that exceed the electron or ion charge by orders of magnitude, and in comparison also have much higher mass-to-charge ratios. Hence, the introduction of very low frequencies characteristic for the charged dust, moving on space and time scales that differ vastly from the normal electron and ion scales [1–6]. Among the many new modes, the prime example is the electrostatic dust-acoustic mode [7], well studied both in theory and in the laboratory [8].

One of the important novel features of dusty plasmas when compared with the usual multispecies electron-ion plasmas, is that the dust particles can interact through both electromagnetic and gravitational forces. When self-gravitational interactions due to the heavier dust component are included, dusty plasmas are subject to a Jeans instability. This results in a significant modification of collective modes and in different stability conditions. In particular, several authors have discussed conditions for the existence of dust-acoustic waves in self-gravitating plasmas [9–13].

Although there have been a number of studies of oblique electrostatic modes in dusty plasmas [14–19], there has been little interest in incorporating self-gravitational effects [20–22]. Many approximations occur in dealing with the low frequencies at which the dust manifests itself, with the small gravitational effects between charged dust particles and with the correct balance between electrostatic and gravitational forces. It is thus necessary to treat the propagation of oblique and perpendicular electrostatic modes in dusty self-gravitational plasmas with the utmost care, as we propose to do in the present paper.

The paper is structured as follows. Section II introduces the basic formalism and the electrostatic dispersion law, without undue approximations, as an extension of our previous work [11]. The low-frequency oblique modes are dealt with in Sec. III, whereas the case of strict perpendicular propagation warrants a separate treatment in Sec. IV. The main results are summarized in Sec. V.

II. BASIC FORMALISM

For simplicity of the subsequent derivations, wave propagation is considered along the z axis, so that $\nabla = \mathbf{e}_z \partial/\partial z$. On the other hand, the static magnetic-field $\mathbf{B}_0$ will be in the x,z plane, with $\mathbf{B}_0 = B_0(\mathbf{e}_x \sin\vartheta + \mathbf{e}_z \cos\vartheta)$, where ϑ is the angle between the directions of wave propagation and the external magnetic field. As pointed out in the Introduction, the basic dusty plasma model includes various species, denoted with running subscripts α. These obey the standard fluid equations, including continuity equations

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial z}(n_\alpha u_{\alpha z}) = 0, \tag{1}$$

and equations of motion,

$$\frac{\partial \mathbf{u}_\alpha}{\partial t} + u_{\alpha z}\frac{\partial \mathbf{u}_\alpha}{\partial z} + \frac{\mathbf{e}_z}{n_\alpha m_\alpha}\frac{\partial p_\alpha}{\partial z} = -\frac{q_\alpha}{m_\alpha}\left(\mathbf{e}_z \frac{\partial \varphi}{\partial z} + \mathbf{u}_\alpha \times \mathbf{B}\right) - \mathbf{e}_z \frac{\partial \psi}{\partial z}. \tag{2}$$

Here n_α, $\mathbf{u}_\alpha$, and p_α refer to the densities, fluid velocities, and pressures, respectively, of the different species, having charges q_α and masses m_α. In this study of electrostatic modes in self-gravitating plasmas, φ denotes the electrostatic potential, wave magnetic fields will be omitted, and ψ is the gravitational potential.

The set of equations is closed by two Poisson's equations, one for the electrostatic potential

$$\varepsilon_0 \frac{\partial^2 \varphi}{\partial z^2} + \sum_\alpha n_\alpha q_\alpha = 0, \tag{3}$$

and one for the gravitational potential

$$\frac{\partial^2 \psi}{\partial z^2} = 4\pi G \sum_\alpha n_\alpha m_\alpha. \tag{4}$$

Standard linear analysis yields for the perturbation densities that

$$n_\alpha = \frac{k^2 N_\alpha}{\mathcal{K}_\alpha}\left(\frac{q_\alpha}{m_\alpha}\varphi + \psi\right), \tag{5}$$

where

$$\mathcal{K}_\alpha = \omega^2 - k^2 v_{T\alpha}^2 - \frac{\omega^2 \Omega_\alpha^2 \sin^2\vartheta}{\omega^2 - \Omega_\alpha^2 \cos^2\vartheta}. \tag{6}$$

Per species N_α represents the equilibrium density, $v_{T\alpha}$ the thermal velocity, and Ω_α the gyrofrequency, including the sign of the respective charge. The expression (5) can be used in both Poisson's Eqs. (3) and (4), yielding then the dispersion law

$$\left(1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\mathcal{K}_\alpha}\right)\left(1 + \sum_\alpha \frac{\omega_{J\alpha}^2}{\mathcal{K}_\alpha}\right) + \left(\sum_\alpha \frac{\omega_{p\alpha}\omega_{J\alpha}}{\mathcal{K}_\alpha}\right)^2 = 0. \tag{7}$$

The plasma frequencies $\omega_{p\alpha}$ are defined through $\omega_{p\alpha}^2 = N_\alpha q_\alpha^2/\varepsilon_0 m_\alpha$, and similarly the Jeans frequencies $\omega_{J\alpha}$ through $\omega_{J\alpha}^2 = 4\pi G N_\alpha m_\alpha$. In the last summation of Eq. (7), $\omega_{p\alpha}$ has to be interpreted as containing the sign of the charge, since $\omega_{p\alpha}\omega_{J\alpha} \propto N_\alpha q_\alpha$.

Proceeding with a first simplification of the dispersion law regardless of frequency regime [11], we not only note that $\omega_{Je}^2 \ll \omega_{pe}^2$ and $\omega_{Ji}^2 \ll \omega_{pi}^2$, but also that

$$\begin{aligned}
\omega_{pi}^2\omega_{Je}^2 &\ll |\omega_{pe}|\omega_{pi}\omega_{Je}\omega_{Ji} \ll \omega_{pe}^2\omega_{Ji}^2, \\
\omega_{pd}^2\omega_{Je}^2 &\ll |\omega_{pe}\omega_{pd}|\omega_{Je}\omega_{Jd} \ll \omega_{pe}^2\omega_{Jd}^2, \\
\omega_{pd}^2\omega_{Ji}^2 &\ll \omega_{pi}|\omega_{pd}|\omega_{Ji}\omega_{Jd} \ll \omega_{pi}^2\omega_{Jd}^2,
\end{aligned} \tag{8}$$

because $\omega_{J\alpha}^2/\omega_{p\alpha}^2 \propto m_\alpha^2/q_\alpha^2$, and in all known dusty plasmas the mass-per-charge ratio for the dust species vastly exceeds that of protons and electrons, in the sense that $m_e \ll m_i \ll m_d e/|q_d|$. The dispersion law (7) is thus to a high degree of precision approximated by

$$\left(1 - \frac{\omega_{pe}^2}{\mathcal{K}_e} - \frac{\omega_{pi}^2}{\mathcal{K}_i}\right)\left(1 + \frac{\omega_{Jd}^2}{\mathcal{K}_d}\right) - \frac{\omega_{pd}^2}{\mathcal{K}_d} - \frac{\omega_{pe}^2\omega_{Ji}^2}{\mathcal{K}_e\mathcal{K}_i} = 0. \tag{9}$$

This dispersion law has to be treated with some care in the low-frequency limit, as discussed in the next sections. Note also that the last term in Eq. (9) cannot automatically be neglected, even though ω_{Ji} is excruciatingly small.

III. LOW-FREQUENCY OBLIQUE MODES

The appropriate low-frequency regime will be defined by asking that $\omega \ll k v_{Ti}$ and $\omega \ll \Omega_i \cos\vartheta$. The latter restriction excludes wave propagation very close to perpendicular, which needs to be treated separately, to be done in the next section. Thus to first order we approximate $\omega_{pi}^2/\mathcal{K}_i \simeq -1/k^2\lambda_{Di}^2$. Analogous results hold even stronger for the electron component, with $\omega_{pe}^2/\mathcal{K}_e \simeq -1/k^2\lambda_{De}^2$. The Debye lengths are defined as $\lambda_{D\alpha} = v_{T\alpha}/\omega_{p\alpha}$, and lead to an effective plasma Debye length λ_D through $\lambda_D^{-2} = \lambda_{De}^{-2} + \lambda_{Di}^{-2}$. Assuming that $k\lambda_D \ll 1$ allows a further simplification of Eq. (9) to

$$\omega^4 - (\Omega_d^2 + \mathcal{L})\omega^2 + \mathcal{L}\Omega_d^2\cos^2\vartheta = 0, \tag{10}$$

where the combination $\mathcal{L} = k^2(c_{da}^2 + v_{Td}^2) - \omega_{Jd}^2$ will determine the stability or instability of the modes, and the dust-acoustic velocity is $c_{da} = \omega_{pd}\lambda_D$. The roots of the biquadratic Eq. (10) are given by

$$\omega_\pm^2 = \frac{1}{2}\left[\Omega_d^2 + \mathcal{L} \pm \sqrt{(\Omega_d^2 + \mathcal{L})^2 - 4\mathcal{L}\Omega_d^2\cos^2\vartheta}\right]. \tag{11}$$

It can be checked that Eq. (10) has two positive roots when $\mathcal{L} > 0$, as then the discriminant, the sum, and the product of the roots are all positive. This is the case in the absence of self gravitation, when $\mathcal{L}$ reduces to $k^2(c_{da}^2 + v_{Td}^2)$. Then ω_+^2 corresponds to the dust cyclotron [19] and ω_-^2 to the dust-acoustic mode, both for oblique propagation with respect to the external magnetic field. For strong magnetization or weak dispersion, such that $k^2(c_{da}^2 + v_{Td}^2)\cos^2\vartheta \ll \Omega_d^2$, we find that

$$\begin{aligned}
\omega_+^2 &\simeq \Omega_d^2 + k^2(c_{da}^2 + v_{Td}^2)\sin^2\vartheta, \\
\omega_-^2 &\simeq \frac{k^2(c_{da}^2 + v_{Td}^2)\cos^2\vartheta}{1 + k^2(c_{da}^2 + v_{Td}^2)(\sin^2\vartheta)/\Omega_d^2},
\end{aligned} \tag{12}$$

which corresponds, but for the inclusion of dust thermal effects, to previous results [19].

As expected from many related physical problems, the presence of self gravitation allows for the possibility of unstable modes. In order to have instability, one needs $\mathcal{L} < 0$, when the product of the roots becomes negative, so that $\omega_-^2 < 0$, while $\omega_+^2 > 0$. The transition to instability occurs at $\mathcal{L} = 0$ and this condition yields the critical Jeans wave number as

$$k_{crit}^2 = \frac{\omega_{Jd}^2}{c_{da}^2 + v_{Td}^2} \simeq \frac{\omega_{Jd}^2}{c_{da}^2}, \tag{13}$$

since usually $v_{Td} \ll c_{da}$. Hence, the plasma condition $k_{crit}^2\lambda_D^2 \ll 1$ is equivalent to $\omega_{Jd}^2 \ll \omega_{pd}^2$.

There are thus three wave-number regimes. For the smallest wave numbers, $\mathcal{L} < 0$ and we find unstable Jeans modes as one of the solutions. For intermediate values of k we have that

$$\omega_{Jd}^2 < k^2(c_{da}^2 + v_{Td}^2) < \Omega_d^2 + \omega_{Jd}^2, \tag{14}$$

so that $0 < \omega_-^2 < \mathcal{L}$, while $\Omega_d^2 < \omega_+^2$. The largest wave numbers obey

$$\Omega_d^2 + \omega_{Jd}^2 < k^2(c_{da}^2 + v_{Td}^2), \tag{15}$$

and then $0 < \omega_-^2 < \Omega_d^2$, while $\mathcal{L} < \omega_+^2$.

For values of k close to the critical values (13), approximate solutions of Eq. (11) can be obtained as

$$\omega_+^2 \simeq \Omega_d^2 + [k^2(c_{da}^2 + v_{Td}^2) - \omega_{Jd}^2]\sin^2\vartheta,$$

$$\omega_-^2 \simeq \frac{[k^2(c_{da}^2 + v_{Td}^2) - \omega_{Jd}^2]\cos^2\vartheta}{1 + [k^2(c_{da}^2 + v_{Td}^2) - \omega_{Jd}^2]\sin^2\vartheta/\Omega_d^2}. \tag{16}$$

The second relation (16) occurs at all angles of propagation, but slowly disappears as perpendicular propagation is approached. The other mode generalizes dust-cyclotron modes. Related investigations were carried out by Salimullah and Shukla [21].

IV. STRICTLY PERPENDICULAR PROPAGATION

For strictly perpendicular propagation $\vartheta = 90°$ and hence $\mathcal{K}_\alpha = \omega^2 - k^2 v_{T\alpha}^2 - \Omega_\alpha^2$. Here, low frequency means that we neglect ω^2 in these expressions, and also assume that $\omega_{Ji}^2/|\mathcal{K}_i| \ll \omega_{Jd}^2/|\mathcal{K}_d|$. It can be checked *a posteriori* that this indeed is fully consistent. Putting

$$A = \left(1 + \frac{\omega_{pe}^2}{k^2 v_{Te}^2 + \Omega_e^2} + \frac{\omega_{pi}^2}{k^2 v_{Ti}^2 + \Omega_i^2}\right)^{-1} \tag{17}$$

allows the rewriting of Eq. (9) as

$$\omega^2 = \Omega_d^2 + A\omega_{pd}^2 + k^2 v_{Td}^2 - \omega_{Jd}^2. \tag{18}$$

Provided $T_i \ll T_e$, we can simplify A to

$$A \simeq \frac{k^2 v_{Ti}^2 + \Omega_i^2}{\omega_{pi}^2}, \tag{19}$$

and hence the dispersion law (18) assumes the form

$$\omega^2 \simeq k^2(c_{da}^2 + v_{Td}^2) + \omega_{dlh}^2 + \Omega_d^2 - \omega_{Jd}^2. \tag{20}$$

The dust lower-hybrid frequency [15] ω_{dlh} has been introduced through

$$\omega_{dlh}^2 = \frac{N_d m_d \Omega_d^2}{N_i m_i} = \Omega_i \Omega_d\left(\frac{N_e}{N_i} - 1\right). \tag{21}$$

Because we have defined all gyrofrequencies with the sign of the charge included, the second expression for ω_{dlh}^2 is correct for negatively as well as for positively charged dust. Furthermore, Eq. (21) indicates that as soon as there is indeed some electron depletion, or conversely, some nonzero dust density, $|\Omega_d| \ll \omega_{dlh}$, and consequently, the Ω_d^2 term can be omitted in Eq. (20). An equivalent way of formulating this condition is to say that most of the mass of the total dusty plasma is in the charged dust $N_i m_i \ll N_d m_d$.

Instability would be possible provided $\omega_{dlh} < \omega_{Jd}$, or equivalently $|\Omega_d| < \omega_{Ji}$, and then at wave numbers obeying

$$k^2 < \frac{\omega_{Jd}^2 - \omega_{dlh}^2}{c_{da}^2 + v_{Td}^2}. \tag{22}$$

This yields smaller wave numbers than previously computed in Eq. (13), and hence implies very large length scales. Besides, the instability criterion (22) actually depends on the sign of the dust charge, through the definition of ω_{dlh}. Indeed, one can expect that for positively charged grains ω_{dlh} could be considerably larger than for negatively charged grains. Other conditions being equal, one can show that

$$\frac{\omega_{dlh(+)}^2}{\omega_{dlh(-)}^2} \simeq \frac{N_{i(-)}}{N_{i(+)}}, \tag{23}$$

a ratio that could be very large. The subscripts $(+)$ or $(-)$ refer to the sign of the dust charge. Thus, plasmas with positively charged dust will be more stable than with negatively charged grains. Only in weakly ionized dusty plasmas ($N_e \sim N_i$) does the dust lower-hybrid frequency drastically drop, and then the criterion (22) turns into Eq. (13).

The magnetic fields that can stabilize the Jeans instability here are given by $B_0 > \omega_{Ji} m_d/|q_d|$. The first impression is that even weak magnetic fields can stabilize the Jeans instability, because ω_{Ji} is so small that it is usually totally neglected. To demonstrate that this is not always true, we rewrite the condition for stability as

$$\frac{\Omega_i^2}{\omega_{pi}^2} > \frac{\omega_{Jd}^2}{\omega_{pd}^2}. \tag{24}$$

In dusty plasmas where $\omega_{Jd}^2 \sim \omega_{pd}^2$, and hence the dust charge-to-mass ratio is quite large, the gravitational collapse can only be stopped by strong magnetic fields, such that $\Omega_i^2 \sim \omega_{pi}^2$.

From previous computations for magnetosonic modes and using the standard probe charging model [5], we can deduce that negative dust charges for micron sized grains in interstellar clouds could be as high as a thousand electron charges [23]. Such estimates are based on very uncertain data about dust in interstellar clouds [24,25], and thus can only be considered as a very rough guidance. Moreover, the resulting Jeans lengths for magnetosonic modes indicated scales from the order of an astronomical unit to several times the dimension of the heliosphere [23]. Other conditions being equal, we can thus conclude for perpendicular electrostatic modes that the influence of the dust lower-hybrid frequency and the possibility of having positively charged dust could substantially increase these Jeans lengths. The unreliability of the present dust data precludes, however, giving any firmer quantitative estimates.

V. CONCLUSIONS

The propagation of oblique and perpendicular electrostatic modes in dusty self-gravitational magnetized plasmas has been studied, with due care for the small gravitational effects between charged dust particles and for the correct balance between electrostatic and gravitational forces. At purely oblique propagation, generalizations of the dust-cyclotron and dust-acoustic modes have been found. The latter can be subject to a Jeans instability, the criterion for which has been given.

For strictly perpendicular propagation, only a mixed dust-acoustic and dust lower-hybrid mode occurs at low frequencies. Generally speaking, the instability criterion becomes dependent on the sign of the dust charge, and plasmas with positively charged dust will be more stable than with negatively charged grains, if other parameters are comparable. A Jeans collapse can be avoided by strong enough magnetic fields, the criterion for which has been given. These results could be of importance when studying the gravitational contraction of large molecular clouds containing charged dust in the presence of magnetic fields, two factors that tend to increase the lengths before instability can set in.

ACKNOWLEDGMENT

F.V. thanks the Bijzonder Onderzoeksfonds of the Universiteit Gent for a research grant.

[1] D.A. Mendis and M. Rosenberg, Annu. Rev. Astron. Astrophys. **32**, 419 (1994).
[2] P. Bliokh, V. Sinitsin, and V. Yaroshenko, *Dusty and Self-gravitational Plasmas in Space* (Kluwer, Dordrecht, 1995).
[3] M. Horányi, Annu. Rev. Astron. Astrophys. **34**, 383 (1996).
[4] F. Verheest, Space Sci. Rev. **77**, 267 (1996).
[5] F. Verheest, *Waves in Dusty Space Plasmas* (Kluwer, Dordrecht, 2000).
[6] P.K. Shukla, Phys. Plasmas **8**, 1791 (2001).
[7] N.N. Rao, P.K. Shukla, and M.Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[8] A. Barkan, R.L. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[9] K. Avinash and P.K. Shukla, Phys. Lett. A **189**, 470 (1994).
[10] B.P. Pandey, K. Avinash, and C.B. Dwivedi, Phys. Rev. E **49**, 5599 (1994).
[11] P. Meuris, F. Verheest, and G.S. Lakhina, Planet. Space Sci. **45**, 449 (1997).
[12] F. Verheest, P.K. Shukla, N.N. Rao, and P. Meuris, J. Plasma Phys. **58**, 163 (1997).
[13] B.P. Pandey and G.S. Lakhina, Pramana **50**, 191 (1998).
[14] L. Mahanta, B.J. Saikia, B.P. Pandey, and S. Bujarbarua, J. Plasma Phys. **55**, 401 (1996).
[15] M. Salimullah, Phys. Lett. A **215**, 296 (1996).
[16] K.D. Tripathi and S.K. Sharma, Phys. Rev. E **53**, 1035 (1996).
[17] H.K. Malik and R. Bharuthram, Phys. Plasmas **5**, 3560 (1998).
[18] H.K. Malik, K.D. Tripathi, and S.K. Sharma, J. Plasma Phys. **60**, 265 (1998).
[19] T. Farid, A.A. Mamun, P.K. Shukla, and A.M. Mirza, Phys. Plasmas **8**, 1529 (2001).
[20] A.A. Mamun, Phys. Plasmas **5**, 3542 (1998).
[21] M. Salimullah and P.K. Shukla, Phys. Plasmas **6**, 686 (1999).
[22] A.A. Mamun, M. Salahuddin, and M. Salimullah, Planet. Space Sci. **47**, 79 (1999).
[23] F. Verheest, M.A. Hellberg, and R.L. Mace, Phys. Plasmas **6**, 279 (1999).
[24] D.C.B. Whittet, *Dust in the Galactic Environment* (Institute of Physics Publishing, Bristol, 1992).
[25] A. Evans, *The Dusty Universe* (Wiley, Chichester, 1994).